Last week I left you with a conditional probability question. Let’s look at its solution now. This will be my last post on GMAT Combinatorics and Probability (for a while at least) until and unless you want me to take up a particular concept/question related to this topic. Next week, we will start a new topic.

Back to question at hand:

Question 2: Alex has five children. He has at least two girls (you do not know which two of her five children are girls). What is the probability that he has at least two boys too? (The probability of having a boy is 0.4 while the probability of having a girl is 0.6)

Solution:

We want to find this probability: P(‘At least 2 Boys and at least 2 Girls’ given ‘At least 2 Girls’) = P(At least 2 Boys and at least 2 Girls)/P(At least 2 Girls)

Let’s try and find P(At least 2 Boys and at least 2 Girls) and P(At least 2 Girls)

‘At least 2 Boys and at least 2 Girls’ can be obtained in two ways: ‘3 Boys and 2 Girls’ or ‘2 Boys and 3 Girls’

P(At least 2 Boys and at least 2 Girls) = P(3 Boys and 2 Girls) + P(2 Boys and 3 Girls)

P(2 Boys and 3 Girls) = 0.4*0.4*0.6*0.6*0.6 * 5!/(2!*3!) = (0.4)^2 * (0.6)^3 * 10

You multiply by 5!/(2!*3!) because out of the five children, any 2 could be boys and the other three would be girls. So you have to account for all arrangements: BBGGG, BGBGG, GGBGB etc

P(3 Boys and 2 Girls) = 0.4*0.4*0.4*0.6*0.6 * 5!/(3!*2!) = (0.4)^3 * (0.6)^2 * 10

P(At least 2 Boys and at least 2 Girls) = [(0.4)^2 * (0.6)^3 * 10] + [(0.4)^3 * (0.6)^2 * 10] = (0.4)^2 * (0.6)^2 *10 (0.6 + 0.4) = (1.6)(0.36)

Now that we have P(At least 2 Boys and at least 2 Girls), let’s focus on getting P(At least 2 Girls). Again, as we saw last week, there are 2 ways of arriving at P(At least 2 Girls).

P(At least 2 Girls) = P(2 Girls and 3 Boys) + P(3 Girls and 2 Boys) + P(4 Girls + 1 Boy) + P(5 Girls)

OR

P(At least 2 Girls) = 1 – P(5 Boys) – P(1 Girl and 4 Boys)

Let me show you the calculations involved in both the methods.

Method 1:

P(At least 2 Girls) = P(2 Girls and 3 Boys) + P(3 Girls and 2 Boys) + P(4 Girls + 1 Boy) + P(5 Girls)

P(2 Girls and 3 Boys) = (0.4)^3 * (0.6)^2 * 10 (from above)

P(3 Girls and 2 Boys) = (0.4)^2 * (0.6)^3 * 10 (from above)

P(4 Girls + 1 Boy) = (0.4)*(0.6) *(0.6)*(0.6)*(0.6)*5!/4! = (0.4) * (0.6)^4 * 5

P(5 Girls) = (0.6)*(0.6)*(0.6)*(0.6)*(0.6) = (0.6)^5

P(At least 2 Girls) = [(0.4)^3 * (0.6)^2 * 10] + [(0.4)^2 * (0.6)^3 * 10] + [(0.4) * (0.6)^4 * 5] + [(0.6)^5]

Method 2:

P(At least 2 Girls) = 1 – P(5 Boys) – P(1 Girl and 4 Boys)

P(5 Boys) = (0.4)* (0.4)* (0.4)* (0.4)* (0.4) = (0.4)^5

P(1 Girl and 4 Boys) = (0.6)* (0.4)*(0.4)*(0.4)*(0.4)*5!/4! = (0.6)*(0.4)^4 * 5

P(At least 2 Girls) = 1 – [(0.4)^5] – [(0.6)*(0.4)^4 * 5]

The values in bold are the same even if they don't look same. (Trust me, I checked on my financial calculator!)

P('At least 2 Boys and at least 2 Girls' given 'At least 2 Girls') = P(At least 2 Boys and at least 2 Girls)/P(At least 2 Girls)

P('At least 2 Boys and at least 2 Girls' given 'At least 2 Girls') = (1.6)(0.36)/[1 – (0.4)^5 - (0.6)*(0.4)^4 * 5]

Even though the solution looks complicated, I hope you see that the approach is quite logical and straight forward. Let's bid farewell to combinatorics and probability now. We will take up some other topic next week. Till then, keep practicing!